1	2	3	4	5	6	Т

1. [4] Consider the motion

$$u_1 = \alpha \sin \frac{2\pi}{l} (X_1 - c_L t) + \beta \cos \frac{2\pi}{l} (X_1 - c_L t), \ u_2 = u_3 = 0$$

in the half-space  $X_1 \ge 0$ . Write expressions for the constants  $\alpha$ ,  $\beta$ , l in terms of all the other parameters assuming the boundary condition in the plane  $X_1 = 0$  is the following:

(a) the displacement is specified,

$$\mathbf{u} = (b\sin\omega t) \mathbf{e}_1$$

Answer:

(a) the surface traction is specified,

$$\mathbf{t}_{ext} = (d\sin\omega t) \, \mathbf{e}_1$$

## Answer:

- 2. [3] Consider a flow of incompressible fluid in a cylindrical tube. The tube diameter is d, the flow rate is G, the density of the fluid is  $\rho$  and the viscosity of the fluid is  $\mu$ .
  - (a) What is the condition to be satisfied by the above parameters so that the flow is laminar? Answer:
  - (b) Assume that d = 15 mm, G = 11/ min, the fluid is water ( $\rho = 1.0 \text{ g/ cm}^3$ ,  $\mu = 1.0 \times 10^{-3} \text{ Pa} \times \text{s}$ ). Determine if the flow in these conditions is or is not laminar and justify the answer on the basis of the condition formulated in the previous paragraph.
- 3. [2] Consider an irrotational flow of an ideal incompressible fluid in the field of a body force **B** given by the expression (in dimensionless variables)  $\mathbf{B} = (9x_1x_2, Ax_1^2, 0)$ , where A is a constant. Determine the value of the constant A.

## Answer \_\_\_\_

4. [2] Potential of an irrotational flow is  $\Phi = x_1^3 - 3x_1x_2^2(x_3^3 - 3x_2^2x_3)$  (in dimensionless variables). Determine the velocity field.

## Answer:

- 5. [4] A cylindrical bar made of steel ( $E_Y = 207 \,\text{GPa}$ ) 3 m long will be designed to withstand a traction force of 445 kN at one of its extremities. What should be the minimum radius of the bar:
  - (a) if the maximum shearing stress should not exceed 100 MPa and the maximum normal stress should not exceed 140 MPa?
  - (b) if it is further required that the elongation should not exceed 0, 12 cm?
- 6. [5] Assume that the air temperature in the atmposphere varies linearly with altitude:  $T = T_0 \alpha x_3$ , where  $T_0$  is ground level temperature and  $x_3$  measures height above the earth. Consider that  $p_0$  is the pressure at ground level, which is known, and that air is an ideal gas. (Recall that the equation of state of an ideal gas can be written as  $p = \frac{R}{M}\rho T$ , where p is hydrostatic pressure, M is the molar mass of the gas which is a known characteristic of each gas,  $\rho$  is the gas density, and R is a given constant.) Determine the air pressure in the atmosphere as a function of  $x_3$  under hydrostatic conditions.